Switched Fuzzy Systems: Overview and Perspectives

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Abstract—**This article surveys recent developments on several fundamental problems for switched systems whose subsystems are all fuzzy systems. It introduces the concept of switch fuzzy systems, their model construction and challenges associated with the stability of such systems. The last includes the issue of stability analysis and finding conditions for existence of switched fuzzy controllers. We also provide motivation for studying these problems by discussing how they arise in connection with various questions of interest in control theory and applications.**

I. Introduction

As computers, digital networks, and embedded systems become ubiquitous and increasingly complex, one needs to understand the coupling between logic-based components and continuous physical systems. This prompted a shift in the standard control paradigm — in which dynamical systems were typically described by differential or difference equations — to allow the modeling, analysis, and design of systems that combine continuous dynamics with discrete logic. This new paradigm is often called *hybrid*, *impulsive*, or *switched control*.

The last couple of decades have witnessed an enormous growth of interest of the class of switched systems in combination with the even larger class of hybrid systems [1][2]. Thus, these systems have a wide range of potential applications. For instance, such systems are widely used in the multiple operating point control systems, the systems of power transmission and distribution, computer disc drives, constrained robotic systems, intelligent vehicle highway systems, etc. Basically, switched systems are special form of hybrid dynamic systems composed of a family of continuous-time or discrete-time subsystems and a rule that governs the switching among them.

From the middle of the 1980's, there have appeared a number of analysis/synthesis problems for Takagi-Sugeno (TS) fuzzy systems. Following the remarkable developments in theory, applications, and the industrial implementations of fuzzy control systems (subway control, autonomous robot navigation, autofocus cameras, image analysis, and diagnosis systems), recently switched systems have been extended further to encompass switched fuzzy systems. The idea was putted forward by Palm and Driankov [3]. Tanaka et.al. [4], [5], [6], on the basis of TS fuzzy systems introduced new switching fuzzy systems for more complicated real systems such as multiple nonlinear systems, switched nonlinear hybrid systems, and second order nonholonomic systems. In general, a switched fuzzy system involves fuzzy systems among its sub-systems or an alternative fuzzy-switching law.

As many nonlinear systems with switching features can be modeled as switching fuzzy systems, it appeared that the class of switched fuzzy systems can describe more precisely both continuous and discrete dynamics as well as their interactions in complex real-world systems. However, the results for switching fuzzy systems in the literature seem to be rather limited, and according to authors it may well be found that there are no studies that survey the research in this area.

The purpose of this study is to shed an innovated light on the latest results on modeling, stability and applications of switched fuzzy systems. In Section 2 we present the basic notions of switched fuzzy systems, related to this study, and continue with an overview on their representation modeling. In Section 3 we outline the latest results on stability analysis of switched fuzzy control systems. Section 4 is devoted to the application areas where switched fuzzy systems can find extensive use. In Section 5 we give the essential conclusions of this study.

II. Concept of Switched Fuzzy Systems and Representation Modeling

There is a vast wealth of literature on switched systems (see for example [1], [2] and the references therein) and fuzzy systems ([7]) alone, but the literature for the unified theory of switched fuzzy systems is barely starting to become available. Thus, rather than provide a complete survey on both of the fields, we provide a discussion of some of the key principles of the joint concept.

There are many different approaches on representation modeling of switched fuzzy systems ([4]-[6], [9]-[29]), but a typical design procedure consists of two steps:

- switched fuzzy model construction for a nonlinear system;
- switched fuzzy controller design.

A lot of research on fuzzy model-based control has been reported. In the fuzzy model-based control, a fuzzy model of a system is constructed exactly or approximately. The complexity of a system makes the number of rules of a fuzzy model exponentially increase. The curse of the number of

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rules makes controller design difficult. To loosen the curse, the authors in [4] proposed a switching fuzzy model. The switching fuzzy model has two key features. One is to switch local Takagi-Sugeno (T-S) fuzzy models represented in each region. The other is to decrease the number of rules which fire simultaneously in comparison with an ordinary fuzzy model. Moreover in [4] a stable fuzzy switching control design is presented. The same idea is extended further in [5] where an application to a hovercraft vehicle as a typical nonholonomic system is given and the switching controller is constructed by naturally extending the idea of the parallel distributed compensation (PDC) [7], [30]. The model in [5] has two levels of structure: region rule level and local fuzzy rule level.

A typical phenomenon observed in switching-based control is non-continuity of control input. It seriously influences important control performance such as ride quality etc. The concept given in [4] and [5] is further extended in [6], where smoothness condition and input constraint for switching fuzzy system are given, which will guarantee smooth switching. The smoothness conditions are also represented via LMIs. A stable switching fuzzy controller satisfying the smoothness condition is designed by simultaneously solving both LMIs.

The idea of switching fuzzy model which consists of local T-S fuzzy models is broadened by introducing the fuzzy model construction based on the sector nonlinearity concept (the approach decreases the number of rules that fire simultaneously in comparison with the ordinary fuzzy model) [8]. In [8] the switching fuzzy model is constructed by dividing the state space into quadrants and by finding the sector which can cover the nonlinear dynamics in each quadrant. In [9], authors showed that the switching fuzzy model with the sectors which tightly cover the dynamics can provide less conservative stability analysis results than that of the ordinary T-S fuzzy models. The conditions are given in terms of bilinear matrix inequalities (BMIs). The same authors in [10] derive LMI controller design conditions based on the switching Lyapunov function by introducing the augmented system which consists of the switching fuzzy model and a stable linear system. As the next step, a switching fuzzy controller design for a class of nonlinear systems is proposed in [11] by the same authors (the PDC idea is employed again).

The switching fuzzy model construction depends on how to divide the state space, which obviously affects stability analysis results. Dividing the state space into quadrants is not always suitable for nonlinear systems. In [12] switching fuzzy model construction with arbitrary switching planes is presented. Moreover, they derive controller design conditions based on the switching Lyapunov function in terms of LMIs for the switching fuzzy model by utilizing the augmented system approach proposed in [11]. The future question that arise is: “How to determine the suitable switching planes for nonlinear systems?”.

All of the works [3]-[6], [8]-[12] have been done just only for the continous-time case. In [13] and [14], contrary to the previous works, two new state-feedback controllers for the discrete-time switching fuzzy system are designed. The first one, associated with a piecewise quadratic Lyapunov function (PQLF), uses time-varying information on the switching-region weighting functions. The second one, associated with a new piecewise fuzzy weighting-dependent Lyapunov function (PFWLF), uses time-varying information on the local fuzzy weighting functions as well as on the switching-region functions.

In [15] and [16] a models of fuzzy systems are proposed, which differs from existing ones. A switching PDC controller is designed for a linear fuzzy system based on switched systems model, i.e. every sub-controller is a PDC controller. Continuous way and discrete way are adopted to establish the stability results.

For better understanding the idea of switched fuzzy system, we address the basic formulation of switched fuzzy system that is taken from [16].

Consider the continuous $T-S$ fuzzy model that involves $N_{\sigma(t)}$ rules of the type as:

$$R^l_{\sigma(t)} : \mathit{If}\ \ \xi_1\ \mathit{is}\ M^l_{\sigma(t)1} \cdots \mathit{and}\ \xi_p\ \mathit{is}\ M^l_{\sigma(t)p},\ \mathit{Then}$$

$$\dot{x} = A_{\sigma(t)l}x(t) + B_{\sigma(t)l}u_{\sigma(t)}(t),\ l = 1,2,...,N_{\sigma(t)}, \quad (1)$$

where $\sigma : R_+ \to M = \{1,2,\cdots,m\}$ is a piecewise constant function and it is representing the *switching* signal. In the rule-based model (1), $R^l_{\sigma(t)}$ denotes the l-th fuzzy inference rule, $N_{\sigma(t)}$ is the number of inference rules, $u(t)$ is the input variable, and the vector $x(t) = [x_1(t)\ x_2(t)\ \cdots\ x_n(t)]^T \in R^n$ represents the state variables. Vector $\xi = [\xi_1\ \xi_2\ \cdots\ \xi_p]$ represents the vector of rule antecedent (premise) variables, with the linear dynamic models in the rule consequents. The matrices $A_{\sigma(t)l} \in R^{n\times n}$ and $B_{\sigma(t)l} \in R^{n\times p}$ are assumed to have the appropriate dimensions.

The i-th fuzzy subsystem can be represented as follows:

$$R^l_i : \mathit{If}\ \ \xi_1\ \mathit{is}\ M^l_{i1} \cdots \mathit{and}\ \xi_p\ \mathit{is}\ M^l_{ip},$$

$$\mathit{Then}\ \ \dot{x}(t) = A_{il}x(t) + B_{il}u_i(t) \quad (2)$$

$$l = 1,2,...,N_i,\ \ i = 1,2,...,m.$$

Thus the global model of the i-th fuzzy sub-system is described by means of the equation

$$\dot{x}(t) = \sum_{l=1}^{N_i} \eta_{il}(\xi(t))\big(A_{il}x(t) + B_{il}u_i(t)\big), \quad (3)$$

together with

$$\eta_{il}(t) = \frac{\prod_{\rho=1}^{n} \mu_{M^l_\rho(t)}}{\sum_{l=1}^{N_i}\prod_{\rho=1}^{n} \mu_{M^l_\rho(t)}},\ \ 0 \le \eta_{il}(t) \le 1,\ \ \sum_{l=1}^{N_i} \eta_{il}(t) = 1, \quad (4)$$

where $\mu_{M^l_\rho(t)}$ denotes the membership function of the fuzzy state variable x_ρ that belongs to the fuzzy set M_ρ^l.

Authors in [17] propose two new relaxed stabilization criteria for discrete-time T–S fuzzy systems. First, they divide the state space into several subregions. Then, the T–S fuzzy system is transformed to an equivalent switching fuzzy system corresponding to each subregion. By means of LMI tools, the local feedback gains of the switching fuzzy controllers are also obtained. The major difference from the previous results is in stability, which is addressed in the next section.

Considerable efforts have been contributed to the problem of designing H∞ fuzzy controllers for a class of nonlinear systems which can be represented by Takagi-Sugeno (T-S) fuzzy models. The problems of designing state feedback H∞ controllers for discrete-time T-S fuzzy systems are given in [18]. A new type of state feedback controllers, namely, switched parallel distributed compensation (PDC) controllers, are proposed, which are switched based on the values of membership functions. The design conditions are given in terms of solvability of a set of linear matrix inequalities (LMIs).

The study of the reliable control design of nonlinear systems, which can tolerate the failure of the control components and maintain the desired system performance, has received significant attention during the last couple of decades. Recently this problem is extended also in the field of switched fuzzy systems. Namely, in [19] the reliable *H*∞ switching fuzzy control problem has been studied for continuous-time nonlinear systems with different actuator failures. First, the nonlinear system is represented or approximated by a T-S fuzzy model. Then, switching fuzzy system is constructed respect to various actuator failures. Next authors present the design of a state-feedback switching controller such that the closed-loop faulty switched system is asymptotically stable with a prescribed *H*∞ performance constraint for arbitrary switching law.

The parameters of many nonlinear plants will change during the operation, e.g., the load of a power system, the number of passengers on board, a train. In these cases, the robustness of the fuzzy controller becomes an important concern. In [21] a systematic analysis and design method is proposed to guarantee the system stability, and design the system performance for a class of nonlinear systems subject to unknown parameters with known bounds. In [22] a fuzzy combined TSK model, which is a fuzzy combination of the global and local fuzzy plant models, is proposed to represent a nonlinear system subject to parameter uncertainties. The global fuzzy plant model is valid to model the full operating domain of the nonlinear plant while the local fuzzy plant model is only valid to model a small domain of the nonlinear plant near the origin of the state space.

In the case of an uncertain fuzzy system, when one controller can not assure the system stability, a control switching strategy in certain set of controllers may stabilize the system. Moreover, the adoption of this kind of strategy can often improve the system performance even in cases a single controller can stabilize the given system. In [23] a model of uncertain fuzzy systems is proposed that differs from existing ones in the literature. A switching PDC controller is designed for an uncertain linear fuzzy system on the grounds of switched systems model, i.e. every sub-controller is a PDC controller. A new model of a class of uncertain switched fuzzy systems is proposed in [25]. A system of this class is a switched system whose subsystems are all fuzzy systems with uncertainties. Continuous controllers for all switched subsystems and a switching law are designed to give robust asymptotic stability. In contrast with the existing results, authors in [25] study switched fuzzy system without levels of structure. The method provides a kind of different premise variable switching directly. In [5], [6], [13], [14] the same premise variable switching with two levels of structure are considered. Authors in [25] design both continuous controllers for subsystems and switching law, while only fixed state-dependent switching is considered in [5], [6], [13], [14]. In [26] the same concept for the model is adopted, where state feedback *H*∞ robust control is studied.

In [27] while exploiting previous results authors are focused on the robust control problem for fuzzy systems due to its considerable importance for practical applications. They consider uncertain switching fuzzy systems and find an alternative solution that overcomes the deficiency of results in the previous works.

Reference [28] studies the problem of reliable control where actuators suffer "serious faults" that tend towards failures, and presents a more general method that combines reliable control and switched T-S fuzzy control. Both fuzzy robust adaptive controllers for subsystems and switching laws which make switched fuzzy system uniformly ultimately bounded are designed. Besides, unknown bounds uncertainties are considered, which are rarely seen in the related literature, and also it is assumed that the bounds of the external disturbances are not necessarily given or approximately known.

In [29] a state feedback H_∞ robust control is studied. In contract with the existing results, here also as in [25] and [26] switched fuzzy system without levels of structure are studied.

III. Stability Analysis and Control Design

Compared with the results on stability of switched systems and those of fuzzy control systems, the results on switched fuzzy systems are very few. Stability analysis of switched fuzzy systems has been pursued mainly based on Lyapunov stability theory but with different Lyapunov functions. One of them is the so-called *common* (or *global*) quadratic Lyapunov functions, another one is the so-called *piecewise* quadratic Lyapunov functions, and the third one is the so-called *fuzzy* (or *non-quadratic*) Lyapunov functions.

In the rest of this section we will present stability analysis results of switched fuzzy systems based on the first two types of Lyapunov functions, as the results of the stability analysis of switched fuzzy systems based on the fuzzy Lyapunov functions are still just a few.

A. Analysis based on common Quadratic Lyapunov functions

One of the first results on stability analysis based on common quadratic Lyapunov functions that are used in switched fuzzy systems is suggested in [4], [5] and since then numerous modifications and improved methods have been proposed. In [4] and [5], authors proposed stabilizing controllers via the "common" quadratic Lyapunov function (CQLF), in which they required that a common positive definite matrix P must be found to satisfy the Lyapunov stability condition for all switching-regions and local fuzzy systems. Although these CQLF-based approaches allowed one to apply convex optimization for solving the synthesis problems, it was mostly found to be conservative because of the "common" (or strict) structure of the Lyapunov function, independent from the switching functions and the local fuzzy weighting functions. In [15], [16] along the innovated representation modeling of continuous-time and discrete-time switched fuzzy systems, and the PDC method for fuzzy controller design, sufficient conditions for asymptotic stability are derived by using the method of single (common) Lyapunov function. Just for illustration we give the definition for quadratic asymptotic stability taken from [16].

Definition 1: [16] The system (1) is said to be quadratic stable if there exist a positive definite matrix P and a state-dependent switching law $\sigma = \sigma(x)$ such that the quadratic Lyapunov function $V(x(t)) = x^T(t)P\,x(t)$ satisfies $\frac{d}{dt}V(x(t)) < 0$ for any $x(t) \neq 0$ along the system state trajectory from arbitrary initial conditions.

It has to be noted that common quadratic Lyapunov functions tend to be conservative, and even worse, might not exist for many complex highly nonlinear systems. This is one of the main limitations of this kind of approaches. Consequently, most of the results for switched fuzzy systems appear dedicated to how to design a fuzzy switching law to achieve asymptotic stability.

In [15], [16] also appropriate stabilizing switching laws in the state-variable dependent form have been synthesized for the cases of continuous and discrete time switched fuzzy systems. Authors in [20] formulate the design problem of the parameters of the switching controller into a LMI problem, whereas the switching scheme is derived under the consideration of system stability. To investigate the stability of the closed-loop system, they use the quadratic Lyapunov function. In [25] design of the robust controller for the control system with the given switching law is also made via the common Lyapunov function approach. Authors in [27] reconstruct the system states by means of observer design and study stability as to ensure its stable asymptotic behavior. The observer error is made to converge to zero under an arbitrary switching law, which has been achieved by using the common Lyapunov function method. The switching law is designed via single Lyapunov function method for the observer such that the overall closed loop control system is guaranteed to be asymptotically stable.

B. Analysis based on piecewise Quadratic Lyapunov functions

Due to the drawback of common quadratic Lyapunov functions, it is thus desirable to develop less conservative stability results for switched fuzzy systems. Piecewise quadratic Lyapunov functions are one of the options available. In order to facilitate development of approaches based on piecewise quadratic Lyapunov functions, one needs partition of the state space.

In [9], authors propose the switching Lyapunov function (the switching Lyapunov function is a class of piecewise Lyapunov function) which consists of local quadratic Lyapunov functions constructed in each region and showed that stability conditions based on the switching Lyapunov function provides less conservative stability analysis results than the ordinary conditions.

The switching Lyapunov function in [9] is defined as:

$$V(x(t)) = \begin{cases} x^T(t)P_1x(t), & x(t) \in R_1, \\ x^T(t)P_2x(t), & x(t) \in R_2, \\ \vdots & \\ x^T(t)P_Qx(t), & x(t) \in R_Q, \end{cases} \tag{5}$$

Where R_q (q = 1,2,…,Q) denotes each region. The stability conditions based on (5) can be found in [9].

In [10], [11], [12] the stabilization conditions to design the switching fuzzy controller based on the switching Lyapunov function are also considered.

In [13] and [14] two new controllers based on a piecewise quadratic Lyapunov function (PQLF) and a new piecewise fuzzy weighting-dependent Lyapunov function (PFWLF) are suggested. The PQLF is largely treated for analysis/synthesis problems of switched hybrid systems, piecewise affine systems, and TS fuzzy systems (but not to the switching fuzzy systems). Hence, applying the PQLF in to the discrete-time switching fuzzy system showed increase in the flexibility and performances of the proposed controllers.

In [17] two new relaxed stabilization criteria for discrete-time T–S fuzzy systems are proposed. First, the state space is divided into several subregions. Then, the T–S fuzzy system is transformed to an equivalent switching fuzzy system corresponding to each subregion. Based on the piecewise Lyapunov function and all possible region transitions between two successive states, the stabilization criteria are derived. In the second criterion, for each subregion the interactions among the fuzzy subsystems in that subregion are represented by one matrix. By this way, the conservatism conduced by representing interactions among the fuzzy subsystems in a single matrix can be reduced. The major differences between the proposed theorems and the previously found in literature are that here instead of multiple Lyapunov function, switching Lyapunov function and switching fuzzy system are used. For the multiple Lyapunov functions, each positive matrix *Pj* is dependent on corresponding rule *j*. For the switching Lyapunov function, the positive matrix *Pj* is corresponding to the switching region.

In [18] switched quadratic Lyapunov functions are exploited to derive a new method for designing switched PDC controllers for guaranteeing the stability and H∞ performances of closed-loop T-S fuzzy systems. The design conditions are given in terms of solvability of a set of linear matrix inequalities (LMIs). It is shown that the new method provides better or at least the same results of the existing design methods via the pure PDC scheme with a quadratic Lyapunov function or switched constant controller gain scheme. Authors in [19] based on the LMI techniques use switching Lyapunov function method for establishing sufficient conditions of existence and design of reliable H∞ switching fuzzy controller.

In the case of an uncertain fuzzy system, when one controller can not assure the system stability, a control switching strategy in certain set of controllers may stabilize the system. Moreover, the adoption of this kind of strategy can often improve the system performance even in cases a single controller can stabilize the given system. In [23], both the single and the multiple Lyapunov function are employed to establish the stability results. Sufficient conditions for asymptotic stability are presented and stabilizing switching laws of the state-dependent form are designed.

Authors in [26] based on the switching strategy, design controller and switching law of the state-dependent form such that the problem of H∞ control is solved. A sufficient condition for stability is given based on the multiple Lyapunov function technique.

IV. Applications of Switched Fuzzy Systems and their Perspectives

Hybrid systems have several high-impact *application areas*, including networked control systems, cooperative control of autonomous systems, communication networks, and systems biology. Consequently switched fuzzy systems can find there application in these areas too.

Here we will address some of the latest applications of switched fuzzy systems that can be found in literature.

Among the earliest applications of switched fuzzy systems is the design problem of fuzzy switching control law for a helicopter, in the whole flight envelope [31]. Here H∞ control approach is considered. For every linearized model in the fuzzy state-space model of a nonlinear system a robust controller is designed. Authors propose a technique for synthesizing the fuzzy switching control law for the helicopter. In [32] the switching method for the fuzzy control of the overhead cranes is presented. One uses the information of the trolley position, load swing and their differentiations to derive the proper control signal. The switching fuzzy algorithm to overcome the condition of dead-zone is especially investigated. Several experiments demonstrate the effectiveness.

One of control methods for underactuated manipulators is known as a switching control which selects a partially-stable controller using a prespecified switching rule. A switching computed torque control with a fuzzy energy region method has been proposed earlier, whereas authors in [33] design some partly stable controllers by the computed torque method, and a switching rule is based on fuzzy energy regions. Design parameters related to boundary curves of fuzzy energy regions are optimized offline by a genetic algorithm (GA). Authors in [34] propose a switching control method based on fuzzy energy regions for this kind of systems, whereas in [36] the same control method is applied to control a two degrees-of-freedom (dofs) underactuated manipulator with one active dof and one passive dof.

The fuzzy logic based supervisor can operate at the highest level of the system and make a switching decision, on the basis of the required performance measure, between two non-linear fixed structure controllers, namely a conventional Proportional-Integral-Derivative (PID) controller, and a PID structure based zero and pole placement controller. The fuzzy supervisor can also adaptively tune the parameters of the controllers. In this context, in [35] a novel fuzzy-logic based switching and tuning supervisor for an intelligent multiplecontroller framework is presented. The proposed methodology is used to simultaneously control the throttle and brake systems of a validated nonlinear vehicle model.

One of the latest, but not the least, applications are found in [37] where a type-2 fuzzy switching control system is proposed for a biped robot, which includes switched nonlinear system modeling, type-2 fuzzy control system design, and a type-2 fuzzy modeling algorithm.

From previously mentioned application areas of switched fuzzy systems, it can be concluded that this systems are ubiquitous and of significant practical application. A unified theory of switched systems is barely starting to become available, which gives us right to expect the later prospect of applications in this area to become widely spread, following and escalating the practical issues that have been solved in the field of switched and fuzzy systems.

V. Conclusion

We have surveyed recent developments in the field of switched fuzzy systems, regarding the modeling, stability analysis, control design and applications. Stability analysis results based on the common and piecewise Lyapunov functions are presented.

For technical details, the reader may consult the references listed below. These references also address many issues that are relevant to switched fuzzy systems but fell outside the scope of this survey. Despite a number of interesting results presented here, it is safe to say that the subject is still largely unexplored.

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